Spirals in PostScript

— *Polar Coordinates and PostScript are mates* —

Abstract

Curves specified in Polar Coordinates can be elegantly programmed in PostScript with the rotate command; which performs rotations in User Space. This has been shown for the Cardioid, the Limaçon, the Lemniscate, the Archimedes and the Growth spiral. The Gyre-logo has been analyzed and imitated in PostScript. Printing of text along spiral-like belts on a sphere in the projection plane has been done, yielding poor man's typesetting text on a sphere in projection.

Keywords

Acrobat Pro, Adobe, art, BLUe, Cardioid, ConTEXt, Cornu, EPSF, Escher, FIFO, folium Descartes, function-grapher previewer, Gyre-logo, hidden lines, IDE (Integrated Development Environment), Jackowski, Lauwerier, Lemniscate, LIFO, Limaçon, Mathematica, MetaPost, Metafont, minimal encapsulated PostScript, minimal mark-up, minimal plain TeX, Photoshop, polar coordinates, projection, PSlib, PSTricks, PSView, rotation of US, Ryćko, Spirals, spherical spiral, TEXworks, Voss.

Prologue

When a curve is specified by Polar Coordinates, $\{\theta, r\}$, the curve is usually drawn by connecting the Cartesian data $(x_i, y_i) = (r_{\theta_i} \cos \theta_i, r_{\theta_i} \sin \theta_i)$, parametric in θ_i.

Folium of Descartes is specified by the equation $x^3 + y^3 + 3axy = 0$.
When I considered how to plot this folium, it was not clear to me how to begin. Using the Cartesian data $(x_i, y_i) = (r_{\theta_i} \cos \theta_i, r_{\theta_i} \sin \theta_i)$, parametric in $\theta_i \in [-30, 120]$, yielded the compact one-sweep drawing.
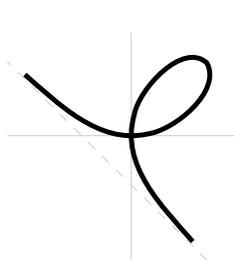

```
%!PS-Adobe-3.0 EPSF-3.0
...
/foliumDescartes{/3a 30 def
/r {3a t sin t cos  mul mul
      t sin dup dup mul mul
      t cos dup dup mul mul add div} def %radius vector
/t -30 def r t cos mul  r t sin mul moveto%start left up
-30  2 120{/t exch def                %to right under
   r t cos mul r t sin mul lineto}for stroke
} bind def
```

Polar Coordinates

$$r_\theta = \frac{3a \sin \theta \cos \theta}{\sin^3 \theta + \cos^3 \theta}$$

Asymptote $x + y + a = 0$.

A curve specified by polar coordinates can sometimes be programmed in PostScript without the calculation of the intermediate Cartesian coordinates, $\{x, y\}$, exploiting the fact that PostScriptperforms rotations in User Space.

Examples A Cardioid is defined in polar coordinates by $r = 2a(1 + \cos(\theta))$, $\theta \in [0, 2\pi]$. A version of the more general Limaçon, with parameter b=.25, $r = 2a(b + \cos(\theta))$, $\theta \in [0, 2\pi]$ is included at right.
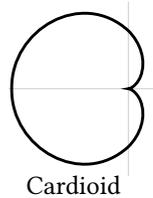
Cardioid

```
%!PS-Adobe-3.0 EPSF-3.0
%%Title: Cardioid in Polar Coordinates
%%BoundingBox: 0-30 -41 30
%%BeginSetup
%%EndSetup
/t 0 def /2a -20 def 2 2a mul 0  moveto
120{3 rotate /t t 3 add def
     2a 1 t cos add mul 0 lineto}repeat
stroke showpage
```
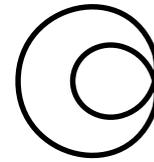

Limaçon

Without the use of the rotation of US facility the coding of the loop would read

```
0 3 360{/theta exch def /r 2a 1 t cos add mul def r theta cos mul r theta sin mul lineto}for
```

The difference is '`r theta cos mul r theta sin mul lineto`' versus '`r 0 lineto`'. Irrelevant details?[1]

The Lemniscate   is defined in polar coordinates by $r^2 = 2a^2 \cos(2\theta), |\theta| \leq \pi/4$, a a parameter.[2]
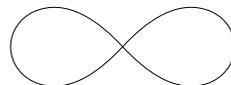
Lemniscate

```
%!PS-Adobe-3.0 EPSF-3.0
%%Title: Lemniscate in Polar Coordinates
%%BoundingBox: -15 -6 15 6
%%BeginSetup
%%EndSetup
/lemniscate{2{gsave -45 rotate /asqrt2 14.142 def  0 0  moveto
-84 6 84{/2t exch def 3 rotate asqrt2 2t cos sqrt mul 0 lineto}for
0 0 lineto stroke grestore -1 1 scale}repeat%left branch mirrored
}bind def
```

In my 'Julia fractals'-paper, EuroTEX2012 I showed that the Cardioid could also have been drawn by Mathematica via `ParametricPlot`. Mathematica has plot variants: `Pot`, `PolarPlot`, `ParametricPlot`, `Plot3D`, `PlotContour`, ...
In Mathematica the Lemniscate can be plotted by `PolarPlot[2 Sqrt[Cos[2t]]`, `{t,0,}$\pi$\type{/4}]`.[3]
My MF code for the Lemniscate of the mid-90s is longer.
In the Archimedes and the Growth spiral the next radius vector is obtained by only one addition casu quo one multiplication.

Stylistic and trigonometric stylistic flower I consider Stylistic and trigonometric stylistic flowers nice examples of using PS' US rotation. I have overlooked the influence `sin` has on the radius vector in Polar Coordinates. At right the picture with $\sin(6\theta)$. I have decoupled the number 6 from the number of petals.
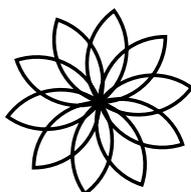
```
%!PS-Adobe-3.0 EPSF-3.0
%%Title: Stylistic flower
%%BoundingBox: -26 -26 26 26
%%BeginSetup
%%EndSetup
/flower{/r 18 def
10 {0 r r 270 360 arc
    r 0 r  90 180 arc
    36 rotate} bind
    repeat stroke
} bind def
```

```
%!PS-Adobe-3.0 EPSF-3.0
0 0 moveto /dtheta 2 def /a 100 def%scaling
10{gsave%one leaf
    dtheta dtheta 30{%points per leaf;
    /6theta exch 6 mul def%30*6=180
    a 6theta sin mul 0 lineto
    dtheta 2 mul rotate%2 thickness leaf
    }for stroke
    grestore
    36 rotate%number of leaves 36*10=360
    }repeat showpage
```
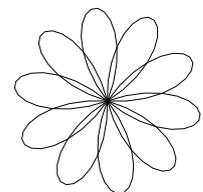

Stylistic flower

Stylistic trig. flower

For Lauwerier's BASIC code of an intriguing trigonometric stylistic flower, and my PS translation, see Appendix 2. Rotation symmetric Julia fractals may also yield stylistic flowers, see Julia Fractals in PS, EuroTEX2012. Stylistic flowers enriched by colours have been shown in my 'Recreational use of TEX', EuroTEX2012.

Introduction

While catching up with font developments in the (TeX) world in the 21st century, I learned about the OTF TeX-Gyre-project for fonts to be used by AllTeX engines. The TeX-Gyre-logo drew my attention, especially the spiral.

In a comment on my 'Recreational use of TeX&Co'-paper Jackowski reminded me of the Toruń1998-logo, which displays a belt with text on a sphere.

Enough for paying attention to Spirals in PostScript.

TeX-Gyre-logo   Toruń1998-logo

Archimedes and Growth Spiral

The Archimedes Spiral, in polar coordinates, is defined by $r_\theta = k\theta$, $0 \leq \theta < \infty$.

The Growth Spiral is in polar coordinates defined by $\ln r_\theta = k\theta$ or $r_\theta = e^{k\theta}$, $-\infty < \theta < \infty$.

If we draw the curves by piecewise straight lines connecting the $\{\theta_i, r_i\}_{i=1}^n$, with θ_i equidistant, then we have the following properties for the discrete radius vectors $\{r_i\}_{i=1}^n$.[4]

Archimedes Spiral: $r_{i+1} = r_i + k\Delta\theta$, $i = 1, ...\infty$, an arithmetic series. The spiral for finite θ_{max} is finite. The grooves are equidistant.


```
%!PS-Adobe-3.0 EPSF-3.0
%%Title: Archimedes Spiral
%%BoundingBox: -56 -59 54 55
%%BeginSetup
%%EndSetup
/archimedesspiral{0 0 moveto
/f .1 def /r 0 def
555{5 rotate /r r f add def
    r 0 lineto}repeat stroke
}bind def
%
archimedesspiral
```

Growth Spiral: $r_{i+1} = r_i\, e^{k\Delta\theta}$, $-\infty < i < \infty$, a geometric series. The spiral for finite θ_{max} is infinite. Rotation yields the same spiral; the spiral is self-similar.


```
%!PS-Adobe-3.0 EPSF-3.0
%%Title: Growth Spiral
%%BoundingBox: -100 -110 75 86
%%BeginSetup
%%EndSetup
/growthspiral{1 0 moveto%off the 0
/f 2.718 .0085 exp def /r 1 def
555{5 rotate /r r f mul def
    r 0 lineto}repeat stroke
}bind def
%
growthspiral
```

Appendix 1 contains Lauwerier's BASIC programs for these spirals using Cartesian coordinates.

My most beautiful Growth spiral is the π-decimals picture. The decimals are printed along an implicit growth spiral because of the scaling and the rotation of User Space. The disk of radius 1 has surface π.

```
%!PS-Adobe-3.0 EPSF-3.0
%%Title: Pi-decimals along a  Spiral, cgl 2012
%%BoundingBox: -80 -100 100 90
%%BeginSetup
%%EndSetup
/Symbol 26 selectfont
1 -18 moveto (p) show%p denotes pi in symbol font
/Helvetica 20 selectfont
0 70 moveto (3) show 1 0 rmoveto (.) show -2 0 rmoveto
-10 rotate .995 dup scale
{pop pop -10 rotate 3 0 rmoveto .995 dup scale}
(3.141592653589793238462643383279502884197169399375\
 ...) kshow showpage
```

More nice Growth spirals appear in the shrinking squares picture.
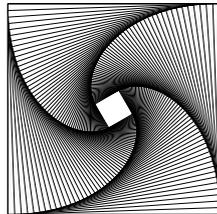

```
%!PS-Adobe-3.0 EPSF-3.0
%%Title: Wervel (HA Lauwerier)
/wervel{/square {-1 s -1 s 2 s  2 s rectstroke} def
 /s {100 mul} def
 /b 12 6.2831 div def
 /c 1 b sin b cos add div def
 64{square b rotate c c scale}repeat
}bind def
```

The Gyre-logo appears to contain a growth spiral. My imitation of the Gyre-logo, with the recursive lattice[5] underneath and with the growth spiral,[6] has been programmed in PostScript as follows.
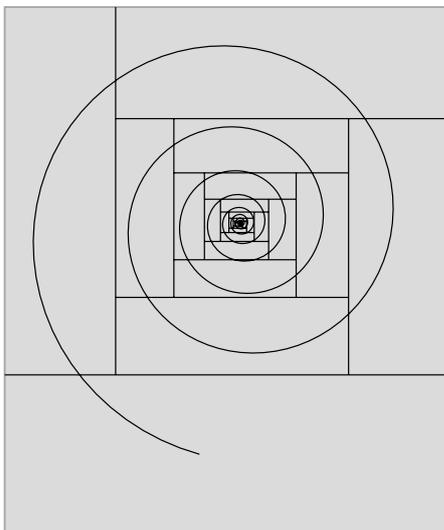

```
%!PS-Adobe-3.0 EPSF-3.0
%%Title: Imitation Gyre-logo, cgl okt 2012
%%BoundingBox: -2 -2 722 862
%%BeginSetup
%%EndSetup
%%BeginProlog
(C:\\PSlib\\PSlib.eps) run % contains growthspiral
%%EndProlog
/ux 720 def /uy 860 def
gsave .86 setgray 0 0 ux uy rectfill
  3 setlinewidth .65 setgray 0 0 ux uy rectstroke%background
grestore
2 setlinewidth
/xl{ux 180 720 div mul}def /xu{ux xl .9 mul sub}def
/yl{uy 260 860 div mul}def /yu{uy yl .7 mul sub}def
7{xl yl moveto xl uy lineto 0 yl moveto  ux yl lineto %low
  xu yl moveto xu yu lineto xl yu moveto ux yu lineto %up
  stroke
  xl yl translate /ux xu xl sub def /uy yu yl sub def
}repeat
%growth spiral
ux uy translate growthspiral showpage
%%EOF
```

Spirals occur naturally in Nautilus shells, sunflower heads, the arms of spiral galaxies, and abundantly in fractals.
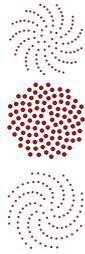
The Nautilus is overlaid with a rectangle with golden-ratio proportions. The rectangle is covered with squares, each time the largest possible. In each square a circular arc has been drawn, yielding the Nautilus 'spiral.'

Cornu's spiral $c(t) = u(t) + iv(t)$, in $\mathbb{C}$, with u and v Fresnel integrals, is famous in Optics for the description of the diffraction of a half-plane. It is also known as a clothoid or Euler's spiral. Advanced, interesting, nice and beautiful.
http://www.mathworld.wolfram/CornuSpiral.html.
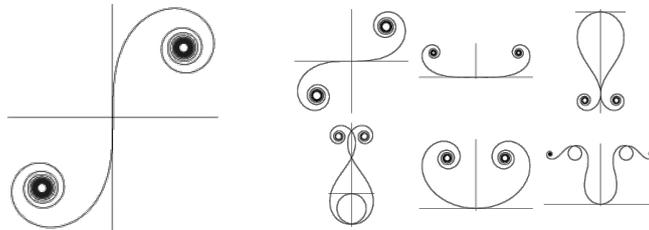
N.M. Temme communicated
a 3D Cornu's spiral from
Temme, N.M(1996): Special Functions
An introduction to Classical Functions
of MathematicaL Physics. John Wiley.
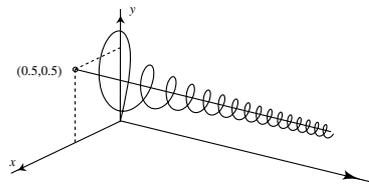
Cornu's spiral
formed from Fresnel integrals
$\{C(t), S(t), t\}$, $t \geq 0$.

The Cornu spiral is the projection of the cork-screw in the (x,y)-plane. At any point of the Cornu spiral the quotient of the curvature and the length equals π, Temme(1996, p185)!

Spherical coordinates
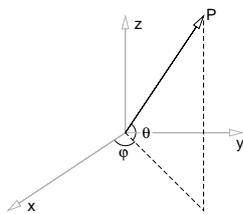
Next to the right-handed XYZ Cartesian coordinate system there is the spherical coordinates system with (r, ϕ, θ), where r is the radius vector, ϕ the azimuth angle and θ the latitude angle. In literature the complement of θ, the angle between the polar axis and the radius vector is also used and called polar angle.
We'll draw the sphere with meridians and latitude circles, the sphere with the spherical growth spiral, and the sphere with belts à la Escher and à la the Toruń-logo, but with text.
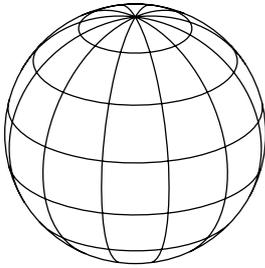
The PostScript codes for the spheres with meridians and latitude circles, ≈ 50 lines — translations of Lauwerier H. A.(1987): Meetkunde met de microcomputer, p110-111 — are too lengthy and too boring to be included here, but ... useful. Consult `PSlib.eps`.

← In PS hidden lines suppressed　　By Mathematica→

`ParametricPlot3D[{Cos[t]Cos[u], Sin[t]Cos[u], Sin[u]}, {t, 0, 2Pi}, {u,-Pi/2,Pi/2}].`

Using Mathematica is simpler and yields more than programming in PS, at expense of a huge `.pdf` file, which makes previewing slow.

Program 50 lines
.pdf 6kB
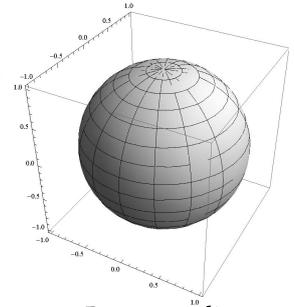
Program 1 line
.pdf 4.5mB, .png 53kB
`Graphics3D[Sphere[]]` 1.2mB

Spherical spiral
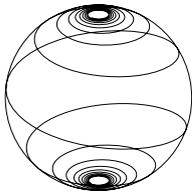
The PostScript code is a translation of `bolspira`, Lauwerier H. A(1987): Fractals, p150.

← spherical growth spiral, hidden lines in view
　　　　spherical growth spiral, hidden lines suppressed→

http://mathworld.wolfram.com/topics/SphericalCurves.html
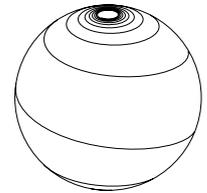
What spiral has been used by Escher and in the Toruń-logo? Apparently not the following logarithmic spherical spiral, because of the ∞ number of windings near the poles.

Below is Lauwerier's BASIC program, and my PostScript translation. Parameters: r = radius sphere, a = density of windings , s=$k\pi/50$, k=-500(1)500, P(x,y,x)=P(r, ϕ, θ) point on the sphere, with ϕ = azimuth, $\theta_k = \mathrm{atan}(a\,s_k)$ = inclination.

```
10 REM ***BOLSPIRAAL, S=phi***
50 A=.15 : REM ***spiraal constante***
60 C=.9  : REM ***helling (uv-) projectievlak***
70 P=1/SQR(2) : Q=P*SQR(1-C*C) : REM ***PROJECTIE
80 FOR N=-500 TO 500          REM    GETALLEN***
90 S=N*PI/50 : T=ATN(A*S)
100 X=COS(S)COS(T) : Y=SIN(S)COS(T) : Z=-SIN(T)
110 U=P*(Y-X) : V=C*Z-Q*(X+Y)
120 IF N=-500 THEN PSET(U,V) ELSE LINE -(U,V)
130 NEXT N : END
```

P(x,y,z)=P(r, ϕ, θ)

$$\begin{aligned}
x &= r\cos\phi\cos\theta \\
y &= r\sin\phi\cos\theta \\
z &= r\sin\theta
\end{aligned}$$

coordinates projection plane

$$\begin{aligned}
u &= (-x+y)/\sqrt{2} \\
v &= -(x+y)/\sqrt{2}\sqrt{(1-c^2)} + cz \\
w &= c(x+y)/\sqrt{2} + z\sqrt{(1-c^2)} \quad \perp uv-plane
\end{aligned}$$

```
/sphereandspiraldict 25 dict def
/sphereandspiral{sphereandspiraldict begin /r 100 def
/c .9 def /a2 1 1.41421 div def
  /b1 a2 1 c c mul sub sqrt mul def
  /c1 c a2 mul def /c3 1 c c mul sub sqrt def
/spiral{90 -1 -90{/thetaj exch def
 /phij thetaj 10 mul phi0 add def%windings
 /x r phij cos thetaj cos mul mul def
 /y r phij sin thetaj cos mul mul def
 /z r thetaj sin mul def
 /w c1 x y add mul c3 z mul add def
 /u a2 y x sub mul def
 /v c z mul b1 x y add mul sub def
 w 0 lt{u v moveto}%hidden lines suppressed
      {u v lineto}ifelse
}for %thetaj
}bind def%spiral
end} bind def
```

Curve on a sphere

I decided to construct a spiral-like curve on the sphere. Below are given: the relation between a spacial point (ϕ, θ) on a sphere of radius r, the coordinates (u,v) in the projection plane and w the coordinate orthogonal to the projection plane.

$$x = r\cos\phi\cos\theta$$
$$y = r\sin\phi\cos\theta \qquad \xrightarrow{\text{projection}}$$
$$z = r\sin\theta$$

$$u = (-x+y)/\sqrt{2}$$
$$v = -(x+y)/\sqrt{2}\sqrt{(1-c^2)} + cz$$
$$w = c(x+y)/\sqrt{2} + z\sqrt{(1-c^2)}$$
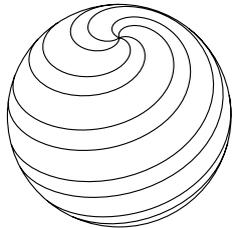
The left sphere has three spiral-like belts with winding constant $k = 4$; parameterized in $t \in [-\pi/2, \pi/2]$ with coordinates

$$x(t) = r\cos 4t\cos t$$
$$y(t) = r\sin 4t\cos t \qquad \text{Escher spherical spirals} \rightarrow$$
$$z(t) = r\sin t$$
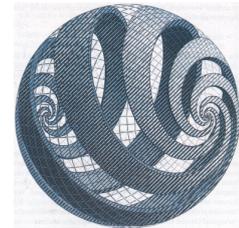
Text on a sphere

I followed the Toruń logo partly by just printing a line of text in the visible part of a belt, by the use of Adobe's pathtext from the Bluebook; no animation nor manipulation of the hidden texts. Too many details.
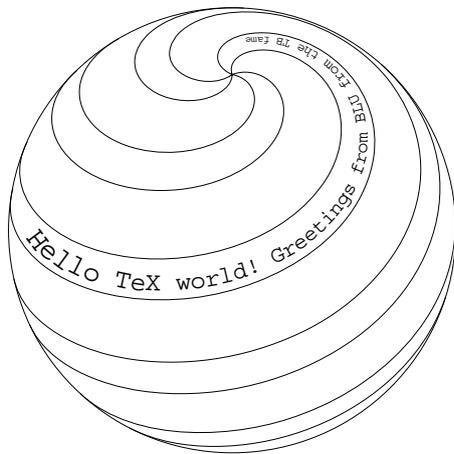
```
/Courier 14 selectfont
/sphereandspiraldict 25 dict def
/sphereandspiral{sphereandspiraldict begin /r 100 def
/c .7 def /a2 1 1.41421 div def /b1 a2 1 c c mul sub sqrt mul def
/c1 c a2 mul def /c3 1 c c mul sub sqrt def
/spiral{-90 1 +90{/thetaj exch def
 /phij thetaj 4 mul phi0 add def%windings
 /x r phij cos thetaj cos mul mul def
 /y r phij sin thetaj cos mul mul def
 /z r thetaj sin mul def
 /w c1 x y add mul c3 z mul add def
 /u a2 y x sub mul def
 /v c z mul b1 x y add mul sub def
 w 0 lt{u v moveto}{u v lineto}ifelse
}for %thetaj
}def%spiral
%
gsave%belts
0 c r mul moveto /phi0   0 def spiral
0 c r mul moveto /phi0  45 def spiral stroke
grestore gsave
0 c r mul moveto /phi0 120 def spiral
0 c r mul moveto /phi0 165 def spiral stroke
grestore gsave
0 c r mul moveto /phi0 240 def spiral stroke
0 c r mul moveto /phi0 285 def spiral stroke
0 c r mul moveto /phi0 275 def spiral
(Hello TeX world, greetings from BLU from the TB fame) 25 pathtext
grestore %circle in projection plane
r 0 moveto
1 1 100{/k exch def  /t k 180 mul 50 div def
        r t cos mul r t sin mul lineto
       }for stroke
end} bind def
```

Shrinking the textsize was achieved by inserting /linetoproc or /pathtext at appropriate places

```
/Courier 14 selectfont /size 14 def
/size size .08 sub def
/Courier findfont [size 0 0 size 0 0] makefont setfont
```

For my spiral-like spherical curves I discretized $\theta_j, j = 90, 89, ...0, ... - 89, -90$ and at the same time I chose for $\phi_j = k\theta_j$ with k a winding constant, the pitch.

If we connect the points $\{\theta_j, \phi_j\}_{j=90}^{-90}$ by straight lines in the projection plane, we obtain the impression of a curve on the sphere. The North and South Poles are $(0, 0, \pm r)$ in Cartesian coordinates. Shifting the curve by rotating the starting position yields a belt on the sphere.

Discussion I found Jackowski's MetaPost code[<reference>] hard to understand.

The various parts are not clearly marked. Moreover, he mentions that he had to kludge around to circumvent limitations, undoubtedly in the parts I have omitted. In PostScript it is straight sailing for the in-principle picture. Jacko's texts are rigid, the characters are fixed in a picture, but the picture shrinks near the poles.

My text is not perfectly typeset either, but it shrinks in size while approaching the poles. Adaptation of textsize is necessary. Hopefully my contribution is also a step forward in the right direction. Much details have to be solved still, maybe for a generation or two after me, with better tools and feedback on the position on the sphere. We could of course inverse (u, v, w) for each character position and obtain the latitude.

Notes Modification of the textsize on the fly is much simpler in PostScript than in TeX&Metafont, although ConTeXt undoubtedly generates fonts on-the-fly.

Experienced readers will discover how to draw the (mirrored) invisible texts, as well as how to manipulate the invisible paths. Animation can be done as well, but requires adjusting of the program.

Herbert Voss I am happy to include Herbert Voss's ribbon.tex; which contains the following ribbon pictures. A wink to LaTeX-PSTricks users. The process-flow is Script$\xrightarrow{\text{TeX}}$ DVI$\xrightarrow{\text{DVIPS}}$ PS$\xrightarrow{\text{Distiller}}$ PDF. The intermediate .ps is 18000 lines, 600kB. I don't know the size of pst-rubans.[7]
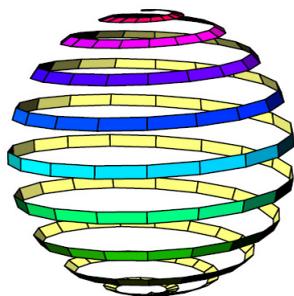


```
\documentclass{article}
\usepackage{pst-rubans}
\begin{document}
\psset{unit=0.75cm}
\begin{pspicture}(-5,-5)(5,5)
\psset{viewpoint=50 20 10,Decran=50,resolution=180}
\psSphericalSpiral[incolor=yellow!50,R=4,hue=0 1,
                   lightsrc=30 5 17]%
\end{pspicture}

\psset{lightsrc=40 25 17}
\psset{unit=0.75,viewpoint=50 30 20,Decran=50}
\begin{pspicture}(-7,-3)(7,8)
\psSpiralParaboloid[incolor=yellow!50,h=6,hue=0 1,
   resolution=360,spires=5,grid,dZ=0.5]
\gridIIID[QZ=3,Zmin=0,Zmax=6](-5,5)(-5,5)
\end{pspicture}
\end{document}
```
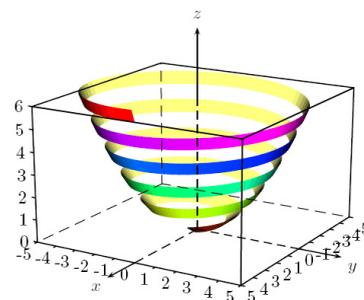
My imitation of Voss' ribbon was achieved by varying the pitch fromfrom 4 into 25, increasing the discretization in θ, reducing the number of belts to one, making the projected circumference gray and changing the viewing angle into /c1 .99 def in the belts-on-the-sphere PS code.
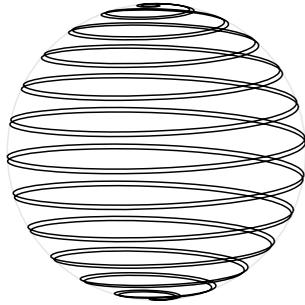
A spherical ribbon as variation of the belts-on-a-sphere. If you want to color it or so, the code is included in `PSlib.eps`

Escher's 'spherical ribbon' →


Tennisball or baseball curve

Different from the spherical spiral but well-known is the curve on the baseball that divides the surface into 2 equal parts.[8] Gabo used the curve for his Spherical object.[9] The four parts of the tennis-ball curve are given by the formulas parameterized in t

$$\{(x, y, z) \mid (1, \sin t, \cos t)\}$$
$$\{(x, y, z) \mid (-1, \sin t, \cos t)\}$$
$$\{(x, y, z) \mid (-\sin t, 1, -\cos t)\}$$
$$\{(x, y, z) \mid (-1, -\sin t, -\cos t)\}$$

$$t \in (-\tfrac{\pi}{2}, \tfrac{\pi}{2})$$
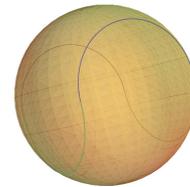
From Wolfram Mathworld Placing Text Randomly.

```
Graphics[Table[
 Inset[Style[Framed[HoldForm[Exp[I Pi] + 1 == 0]], 11],
  RandomReal[{-1, 1}, 2], Center, Automatic,
  RandomReal[{-1, 1}, {2, 2}]], {9}]]
```
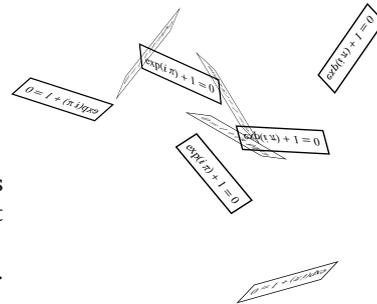

Mathematica, basically a formula-manipulation program. It allows users to arbitrarily place text strings and annotate graphics with text labels. The notebooks can also be formatted, no experience as yet. However … text in a spiral belt on a sphere is decades away, I guess.

However …

Similar to how I post-processed a symbolic flowers, my wife, Svetlana Morozova, used Photoshop to inscribe a sphere with 'Hello TEX world' to create a nice X-mas card.

Acknowledgements

Thank you Adobe for your maintained, adapted to LanguageLevel 3 since 1997, good old, industrial standard PostScript and Acrobat Pro (actually DISTILLER) to view it, as well as pathtext, for typesetting text along an arbitrary path. Don Knuth for your stable plain TEX, Jonathan Kew for the TEXworks IDE, Hàn Thế Thành for pdf(La)TEX,

Thank you Bogusław Jackowski for supplying me with artistic material from GUST, especially sending me the Toruń-logo.

Thank you Herbert Voss for your ribbon-on-sphere pictures made by LATEX-PSTricks.

Thank you Nice Temme for your 3D Cornu's spiral.[10]

Thank you Wim W. Wilhelm, Jos Winnink and Henk Jansen for suggestions and proofing, MAPS editors for improving my use of English and last but not least Taco Hoekwater for procrusting my plain TEX preprint note into MAPS format.
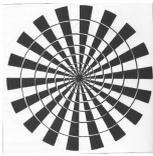
Spiral or ...
Peitgen c.s(2004):
Chaos and Fractals

Conclusion

Drawing curves specified in Polar Coordinates works because PostScriptrotation occurs in User Space. The Gyre-logo has been imitated in PS. Belts of text have been drawn on a sphere. The result is less advanced then that of the Toruń-logo; although the effect of making the textsize dependent on the latitude is nice. A ribbon inscribed on a sphere made a nice X-mas card.

My case rest, have fun and all the best.

Appendix 1: Lauwerier's BASIC codes for the Archimedes and Growth spiral

In Lauwerier(1987): Fractals—meetkundige figuren in eindeloze herhaling, Aramith,[11] the Archimedes spiral and the growth spiral have been coded in BASIC, via the usual Polar and Cartesian coordinates.

```
10 REM ***Spiraal van Archimedes***        10 REM ***Logarithmische Spiraal***
50 A=.1 : PI=3.141593 : PSET (0,0)          50 A=.1 : B=.1 :  PSET(A,0)
60 FOR T=0 T0 16*PI STEP .1: R=A*T           60 FOR T=0.1 T0 35 STEP .1: R=A*EXP(B*T)
70     X=R*COS(T) : Y=R*SIN(T) : LINE -(X,Y)  70     X=R*COS(T) : Y=R*SIN(T) : LINE -(X,Y)
90 NEXT T : END                              90 NEXT T : END
```

Appendix 2: Lauwerier's BASIC code

for the trigonometric stylistic flower
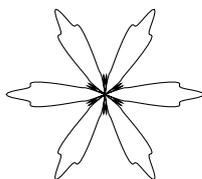

```
 A1=1.5 : A2=.2 : PI=3.141593
S1=6 : S2=36 `parameters
T=0 : H=.01  `initialization
DO WHILE T<2*PI
   r=1+A1*COS(S1*T)+A2*COS(S2*T)
   X=R*COS(T) : Y=R*SIN(T)
   IF T=0 THEN PSET(X,Y) ELSE LINE-(X,Y)
   T=T+H
LOOP END
```

```
%!PS-Adobe-3.0 EPSF-3.0
%%Title: Stylistic flower
%%Formula: r=1+a1*cos(s1*theta)+a1*cos(s2*theta)
%%BoundingBox: -101 -91 101 91
%%BeginSetup
%%EndSetup
/a 35 def%scaling
/a1 1.5 def /a2 .2 def%parameters
/s1 6 def /s2 36 def
/t 0 def /h 1 def %initialization
{ t 360 gt {exit}if
 /r 1 a1 s1 t mul cos mul add
      a2 s2 t mul cos mul add a mul def
 r t cos mul  r t sin  mul t 0 eq{moveto}{lineto}ifelse
 /t t h add def
 }loop closepath
stroke showpage
%%EOF
```

At right my PS translation. Lauwerier, H.A(1992): Computer Simulaties — De wereld als model. Aramith. Lauwerier calls the centre filaments.

Notes

1. During my education I was taught that details matter.
2. In The 8th March note, I drew the Lemniscate in Cartesian coordinates, because the path was reused for printing text along.
3. Abel, M.L, Braselton, J.P, Braselton, L.M(2002): Introduction to Mathematica, p92. Other interesting introductions to Mathematica http://math.georgiasouthern.edu/math/computer/ABGMATHs1., or http://www.science.uva.nl/onderwijs/lesmateriaal/Mathematica/ of 2004 or Wolfram Mathematica Tutorial Collection from 2008 onward. It is amazing how much calculus intelligence has been programmed into Mathematica.
4. Questions about the tangent, the arc length, and the curvature stimulated the development of calculus. For formulas see Wolfgram's Mathworld.
5. The lattice can be understood as: draw the 'cross' within the rectangle and repeat the process within the rectangle of the 'cross'.
6. The centre of the growth spiral is given by the coordinates of the last, small, inner rectangle.
7. Processing TeXnicCentre yielded that pst-rubans was not known, apparently it is not on the TEXlive DVD.
8. Thompson, R.B(1998): Designing a baseball cover. College Math J.
9. Gabo's Torsion, MAPS 42,69–110.
10. Nico used Lauwerier's BASIC 3D drawing program, translated it into PASCAL and obtained .eps from there.
11. Translated into Englisch ISBN: 0-691-8551-x.

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